SHOW YOUR WORK **Name:** _____

1. The urn contains 4 yellow marbles, 5 blue marbles, and 6 red marbles. Two marbles are randomly drawn. What is the probability that both are blue (a) with replacement and (b) without replacement?

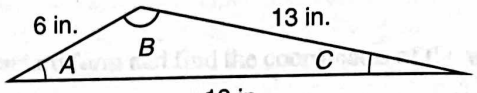
2. A pair of dice is rolled. What is the probability that the number rolled will be less than 6?

3. How soon after four o'clock will the hands of the clock be together again?

4. Sketch the graph of $y = -3 + 4\cos(x + 60°)$.

5. Solve $2\sin^2 3\theta = 11\cos 3\theta + 7$ given that $0° \le \theta < 360°$.

6. Find the measure of the largest angle.



7. Use Cramer's rule to solve for y: $\begin{cases} 3x - 8y = -2 \\ -4x + 5y = 7 \end{cases}$

8. Solve for x: $4^{9x-2} = 9^{4x+3}$

9. Find the distance from the point (4, 1) to the line $4x = 11 - y$.

10. Simplify $\cos\left(\theta + \frac{\pi}{4}\right)$ by using sum and difference identities. Use exact values.

11. Write the four fourth roots of $625 \text{ cis } 120°$ in rectangular coordinates. Give exact answers.

12. Develop the identity for $\tan(A + B)$ by using the identities for $\sin(A + B)$ and $\cos(A + B)$.

13. Solve for x: $\frac{1}{3}\log_4 27 - \log_4(x - 8) + \log_4(x - 1) = \log_4 24$

14. Show: $\frac{\sec x}{1 - \cos x} - \frac{\sec x}{1 + \cos x} = 2\csc^2 x$

15. Two cables are attached to a vertical tower from a point on the ground. The angle between the cables is 30°. The longer cable is 230 feet long and attached to the top of the tower. The shorter cable is attached to the tower 140 feet below the top of the tower. Find the length of the shorter cable.

16. Write the equation in standard form and graph the ellipse: $49x^2 + 4y^2 = 196$

17. The amount of substance initially present was 600 grams, and after 35 hours only 350 grams remained. Assume exponential decay. Write the exponential equation describing the amount of substance present as a function of time and determine the half life of the substance. Sketch the graph.

18. Find the twenty-third term of an arithmetic sequence whose first term is 8 and whose common difference is −5.

19. Write the first three terms of an arithmetic sequence in which the thirty-first term is 34 and the forty-ninth term is −56.

20. Given the hyperbola $x^2 - 9y^2 = 81$, write the equation in standard form and find the coordinates of the vertices and the equations of the asymptotes. Graph the hyperbola.

Test 22 — 11/26/19 — 98 A+

1 4R 5B 6R = 15T

$\frac{5}{15}\cdot\frac{5}{15}-\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{9}$

$\frac{5}{15}\cdot\frac{4}{14}=\frac{1}{3}\cdot\frac{2}{7}=\frac{2}{21}$

2 2 dice < 6

11	21	31	41
12	22	32	
13	23		
14			

4+3+2+1 = 10

$\frac{10}{36}=\frac{5}{18}$

3 $h_L\ T=5$ $h_B\ T=5+20$

$h_L=\frac{1}{12}$ $h_B=1$

$\frac{T}{12}=5$ $T=\frac{T}{12}+20$

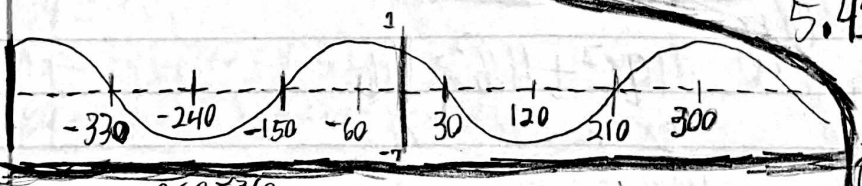
$\frac{12T}{12}-\frac{T}{12}=20$

$\frac{11T}{12}=20$

$T=21.818\text{ min}$

4:22

4 $y=-3+4\cos(x+60)$



$0\le\theta<360$

5 $25\sin^2 3\theta=11\cos 3\theta+7$

$2(1-\cos^2 3\theta)=11\cos 3\theta+7$

$-2+2\cos^2 3\theta+11\cos 3\theta+7=0$

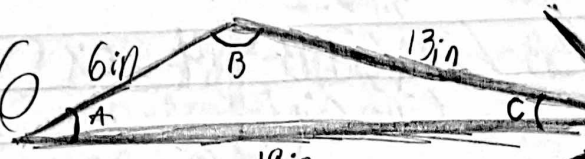
$2\cos^2 3\theta+11\cos 3\theta+5=0$

$(2\cos 3\theta+1)(\cos 3\theta+5)$

$\cos 3\theta=-\frac{1}{2},-5$ $3\theta=120,240,480,600,840,960$

$\theta=40,80,160,200,280,320$

6



$18^2=13^2+6^2-(2)(6)(13)\cos(B)$

$324=169+36-(156)(\cos B)$

$324-205=-(156)(\cos B)$

$119=-156\cos B$

$\cos B=-.7628$

$B=139.71$

7

$3x-8y=-2$

$-4x+5y=7$

$x=\frac{\frac{e}{f}\ \frac{b}{d}}{\frac{a}{c}\ \frac{b}{d}}=\frac{ed-fb}{ad-bc}$ $y=\frac{\frac{a}{c}\ \frac{e}{f}}{\frac{a}{c}\ \frac{b}{d}}=\frac{af-ce}{ad-bc}$

$x=\frac{(-2)(5)-(7)(-8)}{(3)(5)-(-8)(-4)}$ $y=\frac{(3)(7)-(-4)(-2)}{(3)(5)-(-8)(-4)}$

$x=\frac{-10+56}{15-32}$ $y=\frac{21-8}{15-32}$

$x=\frac{46}{-17}$ $y=\frac{13}{-17}$

8 $4^{9x-2}=9^{4x+3}$

$(9x-2)(\log 4)=(4x+3)(\log 9)$

$(9x-2)(.602)=(4x+3)(.954)$

$5.418x-1.204=3.816x+2.862$

$5.418x-3.816x=2.862+1.204$

$1.602x=4.066$

$x=2.538$

9 $4x=\frac{11}{4}-6$

$6=-4x+11$

$6=\frac{1}{4}x+B$

$1=\frac{1}{4}\cdot 4+B$

$1-1=B$

$B=0$

$6=\frac{1}{4}x+0$

$-4x+11=\frac{1}{4}x+0$

$11=\frac{1}{4}x+\frac{16}{4}x$

$11=\frac{17}{4}x$

$x=\frac{44}{17}$ $6=\frac{1}{4}\cdot\frac{44}{17}+0$

$6=\frac{11}{17}$

$4-\frac{44}{17}=\frac{68}{17}-\frac{44}{17}=\frac{24}{17}=x$

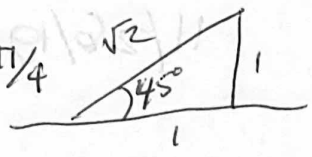
$1-\frac{11}{17}=\frac{17}{17}-\frac{11}{17}=\frac{6}{17}=6$

$\sqrt{\left(\frac{24}{17}\right)^2+\left(\frac{6}{17}\right)^2}$

$\sqrt{\frac{576+36}{289}}=\frac{\sqrt{612}}{17}$

$\frac{+6\sqrt{17}}{-17}$

10 → 20



$\frac{\pi}{4}=45$ $\frac{\pi}{2}=90$ $\pi=180$

$\frac{\pi}{3}=60$ $\frac{\pi}{6}=30$ $2\pi=360$

$\frac{\pi}{4}=90$ NO ↑ $\cos 90=0$ $\sin 90=1$

10 $\cos(A+B)=\cos A\cos B-\sin A\sin B$

2 $\cos(\theta+\frac{\pi}{4})=\cos\theta\cos\frac{\pi}{4}-\sin\theta\sin\frac{\pi}{4}$

$\cos\theta(0)-\sin\theta(1)$

X $-\sin\theta$

11 $(625\text{cis}120)^{\frac{1}{4}}$ $\frac{360}{4}=90$

$5\text{cis}30=5(\cos30+i\sin30)$ $\frac{30}{4}$

$5\text{cis}120=5(\cos120+i\sin120)$

$5\text{cis}210=5(\cos210+i\sin210)$

$5\text{cis}300=5(\cos300+i\sin300)$

$5(\frac{\sqrt3}{2}+\frac{1}{2}i)=\frac{5\sqrt3}{2}+\frac{5}{2}i$

$5(-\frac{1}{2}+\frac{\sqrt3}{2}i)=-\frac{5}{2}+\frac{5\sqrt3}{2}i$

$5(-\frac{\sqrt3}{2}-\frac{1}{2}i)=-\frac{5\sqrt3}{2}-\frac{5}{2}i$

$5(\frac{1}{2}-\frac{\sqrt3}{2}i)=\frac{5}{2}-\frac{5\sqrt3}{2}i$

12 $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$

$\frac{\sin A\cos B}{\cos A\cos B}+\frac{\cos A\sin B}{\cos A\cos B}$

$\frac{\cos A\cos B}{\cos A\cos B}-\frac{\sin A\sin B}{\cos A\cos B}$

$\frac{\tan A+\tan B}{1-\tan A\tan B}$

13 $\frac{1}{3}\log_4 27-\log_4(x-8)+\log_4(x-1)=\log_4 24$

$\log_4(\frac{27^{\frac{1}{3}}}{x-8})(x-1)=\log_4 24$

$(\frac{3}{x-8})(x-1)=24$

$\frac{3x-3}{x-8}=24$

$3x-3=24(x-8)$

$3x-3=24x-192$

$-3+192=24x-3x$

$189=21x$

$x=9$

14 $\frac{\sec x}{1-\cos x}-\frac{\sec x}{1+\cos x}=2\csc^2x$

$\frac{(\sec x)(1+\cos x)-(\sec x)(1-\cos x)}{(1-\cos x)(1+\cos x)}=2\csc^2x$

$\sec x+\frac{\cos x}{\cos x}-\sec x+\frac{\cos x}{\cos x}=\frac{2}{\sin^2x}$

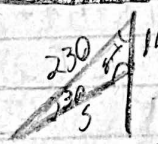
$1-\cos^2x$

$\frac{2}{1-\cos^2x}=\frac{2}{\sin^2x}$

$\frac{2}{\sin^2x}=\frac{2}{\sin^2x}$

$2\csc^2x=2\csc^2x$

15



$\frac{140}{\sin30}=\frac{230}{\sin A}$

$\sin A=\frac{230\sin30}{140}$

$180-55.228=124.772=A$

180.000
−124.772
39.000
$25.228=x$

$\frac{140}{\sin30}=\frac{s}{\sin25.228}$

$\frac{140\sin25.228}{\sin30}=s$

$s=119.342$

16 $49x^2+46^2=196$

$\frac{49x^2}{196}+\frac{46^2}{196}=1$

$\frac{x^2}{4}+\frac{46^2}{49}=1$

$x=\pm2$ $46=\pm7$

17–20 →

17 $A_t = A_0 e^{kt}$

$350 = 600 e^{35k}$

$.583 = e^{35k}$

$-.539 = 35k$

$k = -.0154$

$A_t = 600 e^{-.0154t}$

$300 = 600 e^{-.0154t}$

$.5 = e^{-.0154t}$

$-.693 = -.0154t$

$45 = t$

18 $A_n = A_0 + (n-1)d$

$A_{23} = 8 + (23-1)(-5)$

$A_{23} = 8 + (22)(-5)$

$A_{23} = 8 - 110$

$A_{23} = -102$

19 $A_n = A_0 + (n-1)d$

$34 = A_0 + 30d$

$-56 = A_0 + 48d$

$-90 = 18d$

$d = -5$

$34 = A_0 + 30 \cdot -5$

$34 = A_0 - 150$

$184 = A_0$

$A_n = 184 + (n-1)(-5)$

$184 - 5 = 179 \quad 184 - 10 = 174$

184, 174, 174

20 $x^2 - 9y^2 = 81$

$\frac{x^2}{81} - \frac{y^2}{9} = 1$

$x = \pm 9 \quad y = \pm 3$

$y = \frac{1}{3}x \quad y = -\frac{1}{3}x$

1 hr 38 min